**Noname manuscript No.**

(will be inserted by the editor)

Bootstrapping for Fuzzy Mediation and Moderated-Mediation Analysis

Da Jeong Kang1,†· Hyun Gu Kang1,†· and Jin Hee Yoon2,\*

Received: date / Accepted: date

**Abstract**

1 Department of Mathematics and Statistics, Sejong University, Seoul 05006, South Korea ; {doramisister, (현구메일)}@naver.com

**\***Correspondence: [jin9135@sejong.ac.kr](mailto:jin9135@sejong.ac.kr); Tel.: +82-10-9118-3135

**†** These two authors contributed equally to this work.

**Keywords:**

# 1 Introduction

When describing human behavior, social scientists and behavioral scientists hold that when people are exposed to particular stimuli, they do not instantly react but rather do so through internal organic body.  We have been curious in the process by which one phenomenon influences another because of this. By adding a third variable, they attempted to investigate the causal relationship between the independent and dependent variables and better comprehend their relationship, and it was discovered that the mediator and moderator components of this variable are separated.

A mediator variable is a variable that logically intervenes between independent and dependent variables in the causal relationship and is required to explain why or how.

For instance, client satisfaction will increase as a result of a company's satisfied products. In other words, consumers who are happy with the product will also be satisfied with the company; on the other hand, customers who are unhappy with the product will typically be less satisfied. The variable that describes how this relationship impacts is the mediator variable in this instance where there is a static correlation between product satisfaction and customer satisfaction. In this relationship, it may be inferred that as consumer trust in the the product's manufacturer increases, so does the positive correlation between product satisfaction and customer satisfaction. The purpose of the mediation effect study is to identify a variable that may more effectively explain the relationship that exists in the middle by determining if there is a meaningful influence between these two variables. A moderator variable is a variable that influences the amount and direction of the relationship between independent and dependent variables. The purpose of a moderation effect study is to determine how moderator factors affect the intensity or direction of the relationship between independent and dependent variables. The purpose is to determine if, and under what circumstances, when, or from whom, the relationship between the two variables is weaker or stronger. Researchers in several fields have researched this mediation impact and moderation effect. [] Additionally, studies have been conducted in the past to confirm the relevance of combining mediation effects and moderation effects for qualitative comprehension between variables. [] One example of this is the moderated mediation effect, which will be discussed in this article. The term "moderated mediation effect," was first introduced in 1984 by James & Bret [], refers to the regulation of a particular variable or the intensity of the mediation effect. In other words, the mediation effect is either reinforced or diminished as the value of the moderator variable increases (Jame & Bret, 1984). In Fig. 1, the mediation effect, moderation effect, and moderated mediation effect are represented as a simple model.

The regression-based assessment techniques developed by Sobel (1982), Baron and Kenny (1986), and Aroian and Goodman test methods have all been extensively utilized in recent thesis to conduct mediation analyses. However, the examination of the mediation effect using Baron and Kenny (1986) only establishes if the mediation effect exists or not; it does not establish its statistical significance. Furthermore, in the analytical sequence and judgment process in the case of statistical significance of the mediation effect, the other Sobel, Aroian, and Goodman methodes are not simple , and these methodes have weak statistical power and do not account for measurement mistakes in the study model. The method employing bootstrap has lately been utilized in several studies as a way to address this since it is thought to have limitations in terms of confirming the mediation model and that it is not accurate.

In the meantime, the study of these mediation models was carried out using "precise numbers." However, there are certain statistics that are difficult to convey with precise numbers in reality because they contain ambiguous phrasing. It is simple to communicate language connotations like "some" and "moderate," but it might be challenging to work with precise numerical data. Particularly in the area of social science that deals with psychology, we often come across such ambiguous facts, and in trying to describe them in exact figures, we not only risk losing knowledge but also run into issues. It is true that a precise number cannot accurately capture a person's mental aspect, for instance, when a person's degree of stress is assessed as a variable. Additionally, even though this is stated numerically, each person's assessment scale is unique, so even if the data value is the same, it could really be a different value. As a result, if it is coded as it is, information loss is unavoidable. As a result, it makes sense to describe it as a soft number, like the fuzzy number that Zadeh initially proposed.

Yoon carried out a mediation study based on fuzzy theory in 2020 []. However, there has been no research done on the bootstrap paper using fuzzy mediation and fuzzy moderated-mediation. (부트스트랩 장점 부각) In example, the bootstrap method, which requires millions of resampling operations, has been more popular lately as access has become simpler and computer speed has increased through the statistical software like AMOS. Therefore, in this study, we suggest utilizing the bootstrapping method to examine the fuzzy mediation model and the fuzzy moderated mediation model.

# 2 Simple Mediation Analysis and Fuzzy Mediation Analysis

**2.1. Simple Mediation Analysis**

**2.2. Simple Fuzzy Mediation Analysis**

In this section, we introduce the definition of fuzzy numbers by Zadeh [21] and simple fuzzy mediation models with mediators introduced by Yoon [22].

**2.2.1 Fuzzy numbers.**

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**2.2.2 Simple Fuzzy Mediation Model**

3. Mediation Analysis for Multiple Covariates

In this section, we propose mediation analysis models for multiple covariates with one mediator and multiple mediators.

**3.1. Mediation Analysis for Multiple Covariates with one mediator**

**3.2. Mediation Aanalysis for Multiple Covariates with multiple mediators**

4. Fuzzy Mediation Analysis for Multiple Covariates

**4.1. Fuzzy Mediation Aanalysis for Multiple Covariates with one mediator**

**4.2. Fuzzy Mediation Analysis for Multiple Covariates with multiple mediator**

**4.3. Estimation for Fuzzy Mediation Analysis for Multiple Covariates with mediators**

For the least squares estimation with fuzzy data, a suitable metric is required on the spaces of fuzzy sets. Here, a useful type of metric can be defined via support functions. The support function of any compact convex set is defined as a function  given by for all

where is the (d-1)-dimensional unit sphere in and denotes the scalar product on . Note that for convex and compact the support function is uniquely determined. A metric on a fuzzy number set is defined by the *-*metric on the space of Lebesgue integrable

Based on this, an *-* metric for fuzzy numbers can be defined by

A fuzzy regression model which was introduced in author’s previous studies [24,25] that is proposed as follows:

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The variables are represented by and for It is assumed that are the fuzzy random errors for expressing fuzziness. Note that we can encompass all cases by

where are the left and right spreads of respectively. Now the estimators is obtained if we minimize following objective function:

for *k=1,2,…,q*, where *q* is the number of the regression model in this fuzzy mediation analysis. And the objective function can be obtained based on the *-*metric, and here the *-* distance can be expressed as follows:

To minimize above equation, we obtain the normal equation applying

And, for each the normal equation, which has as solutions, can be obtained as follows:

To find the solution vector, we define a *triangular fuzzy matrix* *(t.f.m.)* which is expressed by

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and denoted by in short, where is a triangular fuzzy number for And we define a triangular fuzzy vector

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To minimize the above objective function, fuzzy operations fuzzy numbers and estimators which were defined in our previous studies [26-29] have been applied.

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For given two *t.f.m*'s, , , and a crisp matrix the operations are defined as follows:

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Using the above operations and algebraic properties, the solutions of normal equation fuzzy estimators are derived for each by

where

and , for Note that (16) exists if .

5. Statistical inferences of Mediation and Fuzzy Mediation Model for Multiple Covariates

This section proposes statistical inferences such as confidence interval and test statistics for the proposed models in section 3 and section 4.

**5.1 Inferences on the total and direct effect**

**5.2. Test statistics for fuzzy mediation model with multiple covariates**

6. Fuzzy Mediation Analysis with Multiple Covariates for Solar Power Data

**6.1 Estimation of total effect, direct effect, and indirect effect**

7. Conclusions

**Acknowledgement** This work was supported by the National Research Foundation of Korea(NRF) grant funded by the Korea government(MSIT) (No. 2020R1A2C1A01011131; No. 2019M3F2A1073179).

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